Calc AB Ch 2A Test Practice

Name: Block: Seat:

Chapter 2A skills Check List:

1 ...... Average Rate of Change versus Instantaneous Rate of Change (secant line slope vs tangent line slope)

2 ...... Conditions for Continuity:

   i. $f(c)$ exists

   ii. $\lim_{x \to c} f(x)$exits (one sided limits must agree)

   iii. $\lim_{x \to c} f(x) = f(c)$

3 ...... Conditions for Differentiability:

   i. Continuous at $f(c)$

   ii. $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ exits (one sided limits must agree)

   iii. not vertical (no slope)

4 ...... Both limit definitions of Derivative (p 103 and 105)

5 ...... Differentiable at a point or on an open interval (p. 103)

6 ...... Differentiability Implies Continuity (p 106)

7 ...... The Constant Rule (p 110)

8 ...... The Power Rule (p 111)

9 ...... The Constant Multiple Rule (p 113)

10 ...... Sum and Difference Rules (p 114)

11 ...... Derivatives of Sine and Cosine (p 115)

12 ...... Product & Quotient Rule (p 122, 124)

13 ...... Derivatives of tan, cot, sec, csc (p 126- if not by heart, by using sin and cos with quotient rule)

14 ...... Higher Order Derivatives (p 128) like velocity and acceleration

15 ...... Chain Rule (p 134)

16 ...... General Power Rule (p 134)

17 ...... Summary of Differentiation Rules (p 139)

18 ...... Derivative of e^x is e^x

19 ...... Derivative of $\ln x$ is $\dfrac{1}{x}$

20 ...... Guidelines for Implicit Differentiation (p 145)

Delta Math Check List:

1 ...... Practice Limit Definition of Derivative (4 types of question)

2 ...... Practice Power Rule (3 types of question)

3 ...... Practice Product and Quotient Rule (3 types of question)

4 ...... Practice Basic Chain Rule (5 types of question)

5 ...... Practice Basic Derivative Questions (5 types of question)

6 ...... Practice Implicit Differentiation (6 types of question)

Khan Academy Check List:

1 ...... AP Calculus AB Unit: **Differentiation: definition and basic derivative rules** (Start the unit test and collect up to 2,300 possible Mastery points each time it is 23 questions and should take 46 minute or less)

2 ...... In the AP Calculus AB Unit: **Differentiation: composite, implicit, and inverse functions** are some chapter 2 topics:

   i. Chain Rule Intro (collect 80-100 Mastery Points)

   ii. Chain rule with Tables (collect 80-100 Mastery Points)

   iii. Implicit Differentiation (collect 80-100 Mastery Points)

1. Definition of Derivative (2.1)

(a) This table gives select values of the differentiable function f.

x	4	5	6	7
$f(x)$	1	18	35	53

What is the best estimate for $f'(7)$ we can make based on this table?

A. 9.6
B. 18
C. 53
D. 11

(b) What is the average rate of change of $g(x) = 7 - 8x$ over the interval $[3, 10]$?

(c) Use the limit Definition of a derivative to show to show the derivative of $f(x) = 3x^2 - 4x$ is $6x - 4$

(d) Evaluate $\lim\limits_{h \to 0} \dfrac{(3+h)^{23} - 3^{23}}{h}$

(e) Evaluate $\lim\limits_{x \to 2} \dfrac{4x^3 - 32}{x - 2}$

(f) Let $g(x) = \ln x$. Which of the following is equal to $g'(5)$?

A. $\lim\limits_{x \to 5} \dfrac{\ln(5+x) - \ln(5)}{x-5}$
B. $\lim\limits_{x \to 5} \dfrac{\ln(x) - 5}{x-5}$
C. $\lim\limits_{x \to 5} \dfrac{\ln(x-5)}{x-5}$
D. $\lim\limits_{x \to 5} \dfrac{\ln(x) - \ln(5)}{x-5}$

2. Continuity and Differentiability (2.1)

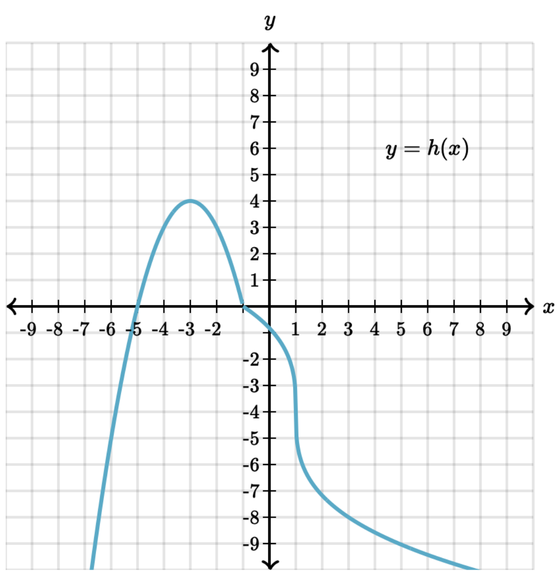
   (a) Let f be the function
   $$f(x) = \begin{cases} x^2 - 1, & x \le 2 \\ 5x - 7, & x > 2 \end{cases}$$
   Which of the following statements about f are true?

   I. f has a limit at $x = 2$
   II. f is continuous at $x = 2$
   III. f is differentiable at $x = 2$

   (b) Let f be the function
   $$f(x) = \begin{cases} -1.5x^2, & x \le -2 \\ 6x + 6, & x > -2 \end{cases}$$
   Which of the following statements about f are true?

   A. Continuous but not differentiable
   B. Differentiable but not continuous
   C. Both continuous and differentiable
   D. Neither continuous nor differentiable

Function h is graphed. The graph has a vertical tangent at $x = 1$.



Select all the x-values for which h is not differentiable.

3. Power / Product / Quotient / Chain Rule Practice (2.2, 2.3, 2.4)

   (a) $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt[4]{x^5}}\right)$

   (b) $\dfrac{d}{dx}\left(\dfrac{1}{x^2} + \dfrac{1}{x} + x\right)$

   (c) $f(x) = (x^2 - 3x + 8)^3$

   (d) $g(x) = (8x - 7)^{-5}$

   (e) $f(x) = \dfrac{x}{(x^2 - 1)^4}$

   (f) $F(v) = (17v - 5)^{1000}$

   (g) $k(r) = \sqrt[3]{8r^3 + 27}$

(h) $H(x) = \dfrac{2x+3}{\sqrt{4x^2+9}}$

(i) $f(\theta) = \dfrac{\sin\theta}{\theta}$

(j) $g(t) = t^3 \sin t$

(k) $h(z) = \dfrac{1-\cos z}{1+\cos z}$

(l) $f(x) = \dfrac{\tan x}{1+x^2}$

(m) $k(x) = \sin(x^2+2)$

(n) $H(\theta) = \cos^5 3\theta$

(o) $g(z) = \sec(2z+1)^2$

(p) $f(x) = \cos(3x)^2 + \cos^2 3x$

(q) $K(z) = z^2 \cot 5z$

(r) $h(\theta) = \tan^2\theta \sec^3\theta$

(s) $h(w) = \dfrac{\cos 4w}{1 - \sin 4w}$

(t) $f(x) = \tan^3 2x - \sec^3 2x$

(u) $f(x) = \sin\sqrt{x} + \sqrt{\sin x}$

(v) $g(x) = \dfrac{e^x}{x}$

(w) $h(x) = x \ln x$

(x) This table gives select values of functions g and h, and their derivatives g' and h', for $x = -4$

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
-4	2	3	-1	5

Evaluate $\dfrac{d}{dx}\left(g(x) \cdot h(x)\right)$ at $x = -4$.

(y) Let $k(x) = f(g(x))$.
If $f(2) = -4$, $g(2) = 2$, $f'(2) = 3$, $g'(2) = 5$, find $k(2)$, $k'(2)$, and the equation of the tangent line of k when $x = 2$.

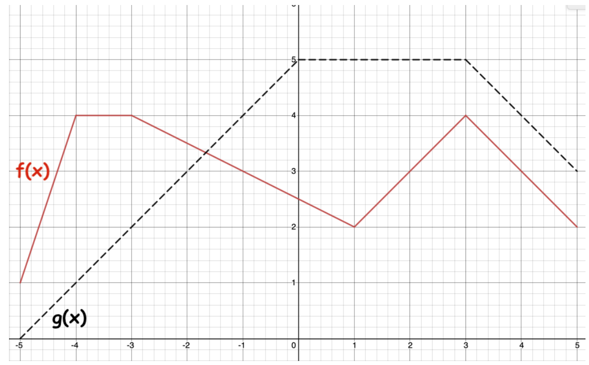
(z) This table gives select values of functions g and h, and their derivatives g' and h', for $x = 3$

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
3	7	-2	8	4

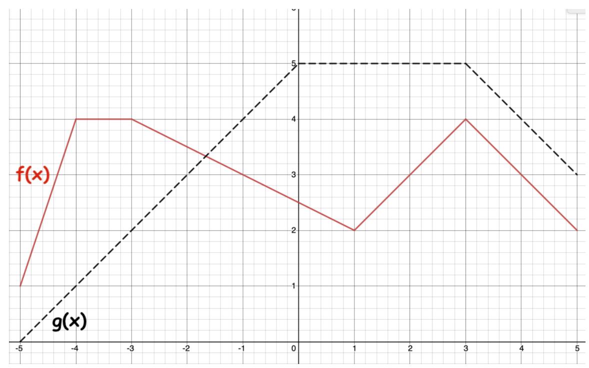
Evaluate $\dfrac{d}{dx}\left(\dfrac{g(x)}{h(x)}\right)$ at $x = 3$.

4. Differentiation Using Graphs
   (AP style of question)

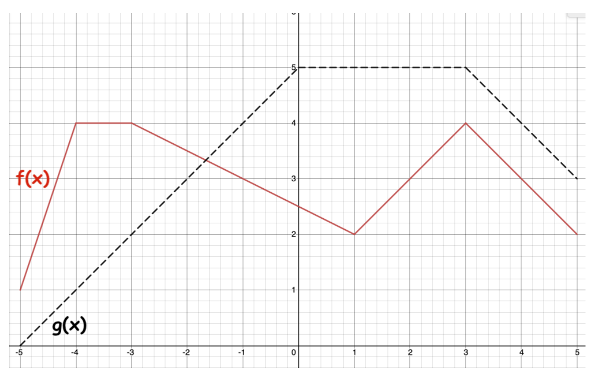
   (a) Given the graph of f and g to the right, let $h(x) = f(x) \cdot g(x)$. Find $h'(-1)$



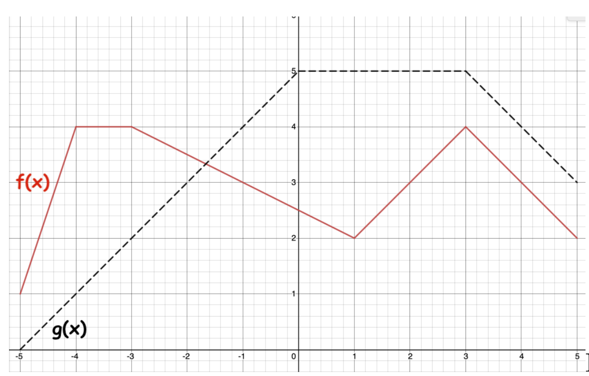
   (b) Given the graph of f and g to the right, let $k(x) = \dfrac{f(x)}{g(x)}$. Find $k'(2)$



   (c) Given the graph of f and g to the right, let $v(x) = f(g(x))$. Find $v'(-1.5)$



   (d) Given the graph of f and g to the right, let $q(x) = g(g(x))$. Find $q'(-1)$



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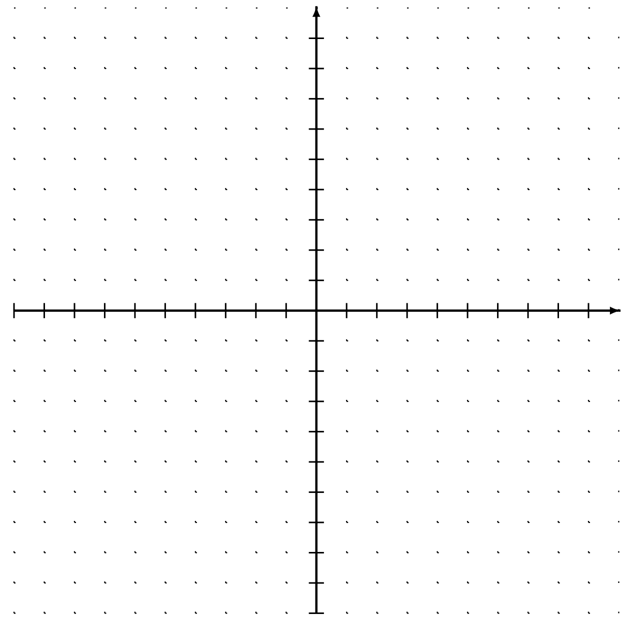
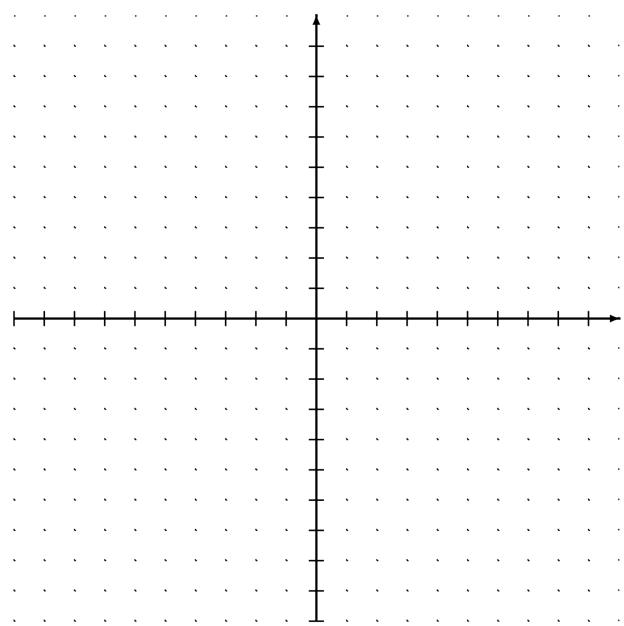
5. Sketch the graph of each function, given the provided information.

   (a) $f(-4) = 3$, $f'(-1) = 0$, $f'(2) = 0$,
   $f'(x) > 0$ for $-4 < x < -1$,
   $f'(x) < 0$ for $-1 < x < 2$,
   $f'(x) > 0$ for $x > 2$,



   (b) $g(0) = 0$, $g'(0) = 0$, $g'(-2) = 0$, $g'(4) = 0$
   $g'(x) > 0$ for $x \geq 0$,
   $g'(x) < 0$ for $x < -2$,
   $g'(x) > 0$ for $-2 < x < 0$,



6. Higher order derivatives (2.3)

   (a) Find the second derivative of the function: $f(x) = (2x^4 + 8)^4$

   (b) Let s be the position (distance) function of a free falling object. Let s be defined to be

   $$s(t) = -4.9t^2 + 120t + 45.$$

   Find the velocity and acceleration of the object when it is 20 meters high. *(Assume the object was projected into the air at the time $t = 0$).*

   (c) Let $f(x) = x^8$.
   Find $f''(x)$, $f^{(8)}(x)$, and $f^{(9)}(x)$

   (d) Let $y = \frac{1}{x}$. Find $\frac{d^3y}{dx^3}$

   (e) Let $y = 2e^{4x}$. Find $\frac{d^2y}{dx^2}$

   A. $32e^{4x}$
   B. $8e^{x}$
   C. $40e^{6x}$
   D. $\frac{e^{4x}}{8}$
   E. $32x^2e^{4x}$

   (f) $h(x) = 6\ln(4x)$. Find $h''(x)$

7. Implicit Differentiation (2.5)

(a) Find $\frac{dy}{dx}$ by implicit differentiation:

$$x^4 + 10x + 7xy - y^3 = 16$$

(b) $2y^2 - x^2 + x^3y = 2$. Find $\frac{dy}{dx}$.

A. $\frac{2x - 3x^2y}{4y + x^3}$

B. $\frac{2x}{4y + 3x^2}$

C. $\frac{2x - 4y}{3x^2}$

D. $\frac{4y + x^3}{2x - 3x^2y}$

(c) Let $y^4 + 5x = 11$. Find $\frac{d^2}{dx^2}$ at the point $(2, 1)$

(d) $3y^2 + x^2 - xy = \pi$. Find $\frac{dy}{dx}$.

A. $\frac{y - 2x}{6y + x}$

B. $\frac{1 - 2x}{6y + 1}$

C. $\frac{y - 2x}{6y - x}$

D. $\frac{1 - 2x}{6y - 1}$

(e) $4x - x^2y + y^3 = 10$ Find the value of $\frac{dy}{dx}$ at the point $(1, 2)$

(f) (challenge)
Let $xy = 18$. Find $\frac{dx}{dt}$ when $x = 2$ and $\frac{dy}{dt} = -6$.

8. Tangent Lines (an application of Derivatives)

   (a) The tangent line to the graph of function f at the point $(5, 7)$ passes through the point $(1, -1)$. Find $f'(5)$

   (b) Let $y = \dfrac{1-2x}{3x^2}$ What is the equation of the tangent line at $(1, -\frac{1}{3})$?

   (c) Let $y = -x^3 + 4x^2$ What is the equation of the tangent line at the point where $x = 3$?

   (d) Let $y = \cot(x)$ What is the equation of the tangent line at $x = \frac{\pi}{6}$?

   (e) $x + 2xy - y^2 = 2$. Find the slope of the tangent line at the point $(2, 4)$.

      A. $\frac{3}{2}$
      B. $\frac{9}{4}$
      C. $\frac{1}{2}$
      D. $-\frac{9}{4}$

   (f) Use implicit differentiation to find an equation of the tangent line to the ellipse

$$\frac{x^2}{2} + \frac{y^2}{98} = 1$$

   at the point$(1, 7)$